should be acceptable for small times. For large times, Eq. (21) yields $q_1 = 4.47(kt/c_v)^{1/2}$, an inordinately large value. However, for large times, a parabolic profile is a better approximation as we demonstrated earlier, so that solution (3) should be used for large times. For intermediate times, either Eq. (15) or Eq. (21) may be chosen. A precise criterion for selecting the appropriate transition point is beyond the scope of this paper. If both equations are used, that which leads to the lower value of q_1 is preferable, since both methods generally overestimate the exact value. Generally speaking, for smaller intermediate times, Eq. (21) should be used, whereas for larger intermediate times, Eq. (15) is preferable.

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Shock Reflection and Interaction at Hypersonic Speeds

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THIS note presents the results of a real-gas (air) calculation of the interaction of hypersonic oblique shock waves. The Mach number-altitude relationship used is representative of possible hypersonic ramjet flight capabilities. Detailed results for a range of wedge angles and Mach numbers have been obtained[1] for the dynamic and thermodynamic variables during and after the shock interaction and for the geometry of the interaction. Emphasis is placed upon conditions considered suitable for the use of the double oblique shock system as an intake for hypersonic ramjets which use supersonic combustion.

Mach [2] showed that, if a shock in an inviscid supersonic stream impinges on a plane wall, two flow geometries are possible, regular reflection and Mach reflection. Regular reflection occurs when an approaching flow is turned toward the wall by the incident shock and is then turned back by the reflected shock to the freestream direction. For Mach reflection, the incident shock must be stronger, thereby turning the flow more sharply toward the wall. If the deflection required to bring this flow to the freestream direction is larger than a critical angle,[3] then the reflected shock will tend to detach. However, detachment cannot occur because of the incident shock's presence; instead a three-shock configuration appears. A normal shock extends some distance up from the surface; it then divides into an incident and a reflected shock. The flow behind the normal shock is subsonic, whereas that behind the reflected shock is supersonic. These two flow regions are separated by a shear layer originating at the intersection of the three shocks.

For inviscid flow, the wall may be considered to be a streamline of symmetry, in which case a mirror image of either of the preceding types of reflections may be thought of as occurring below this line. In this case, the shocks are interacting instead of reflecting. Again there exist two possible flow geometries, regular interaction and Mach interaction. An important difference between reflection and interaction of shocks is that the latter is free from the viscous effects of boundary layers. Tepe and Tabakoff[4] consider the wedge boundary-layer effect on interaction of unequal shocks for an ideal gas from Mach 6 to 20. The flow displacement due to the shear layer has not been considered.

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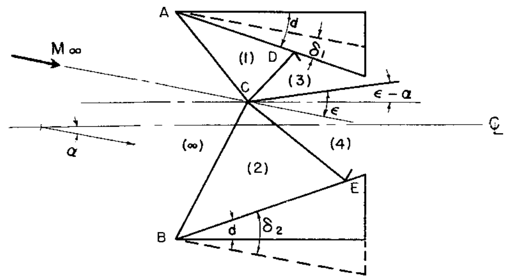

Fig. 1 Double-wedge intake with two interacting oblique shock waves.

The interaction of unequal shocks may be treated as a reflection problem, with the streamline through the interaction point no longer symmetrical, but deflected from the upstream flow by an angle ϵ. This deflected streamline separates two uniform flow regions that have undergone compressions through different shock systems but still retain identical flow directions and static pressures. The flow speed discontinuity gives rise to a high vorticity layer known as a contact surface or a slipline.

If two equal wedges of angle d are at an angle of attack α to the freestream flow, then the two wedges deflect the flow by unequal amounts. This is equivalent to having no angle of attack and the equivalent aerodynamic angles

$$\delta_1 = d - \alpha \qquad \delta_2 = d + \alpha \tag{1}$$

This regularly interacting shock system may be considered to act as a simple air intake for air-breathing propulsion systems.

Freestream conditions[5] were taken along the "boost path,"[6] which is a Mach number-altitude relationship, representative of possible hypersonic ramjet flight capabilities. The oblique shock characteristics across the first shocks are calculated by a method similar to Ref. 7 but using only one iteration. The conditions in regions 3 and 4 (Fig. 1) after the second shocks are obtained by requiring matched flow directions and equal static pressures. These conditions follow from the continuity and momentum equations, by considering the slipline as a streamline. The equivalent deflection angles for regions 3 and 4, i.e., the changes in flow direction before and after the second shocks, are

$$\delta_3 = \delta_1 + \epsilon \qquad \delta_4 = \delta_2 - \epsilon \tag{2}$$

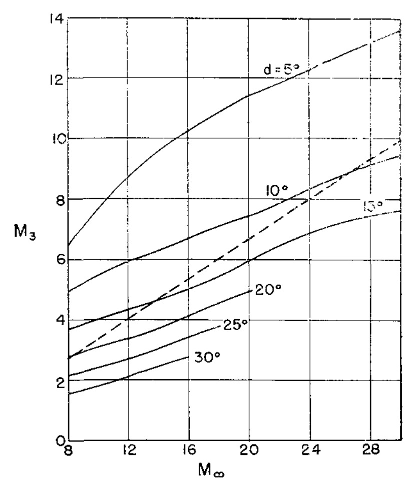

Fig. 2 M_3 vs M_∞ and d for $\alpha = 0°$.

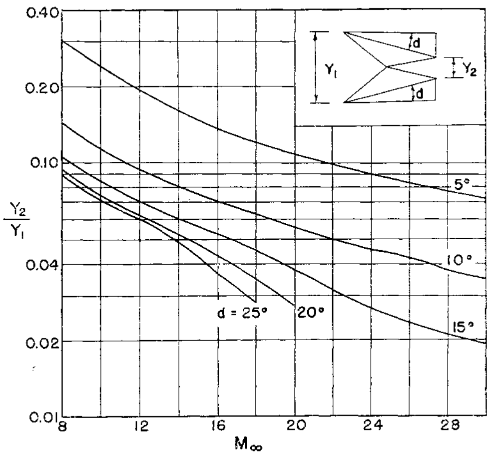

Fig. 3 Area ratio vs M_∞ and d for $\alpha = 0°$.

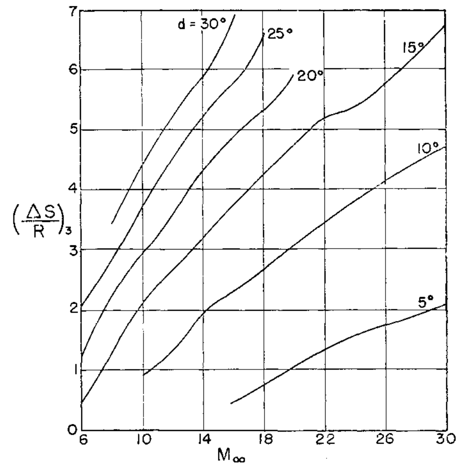

Fig. 5 Nondimensional entropy increase vs M_∞ and d for $\alpha = 0°$.

A double iteration procedure, involving both ϵ and the oblique shock characteristics, is used to calculate the conditions after the second shocks. Pressures across the contact surface were matched to five significant figures. Boundary-layer effects were not considered, since the deflection angles (δ) were treated as the deflections of the fluid and not the physical deflections of the wedges.

Reference 8 shows that optimum hypersonic ramjet performance is obtained with an intake which reduces the free-stream Mach number by a factor near three. In Fig. 2, this optimum line is shown dashed, and it is seen that, for optimum intake performance over a Mach number range of 8 to 26, it is the flow deflection (i.e., wedge angles) that should vary from 15° to 10°. The compression ratio (p_3/p_∞) of such a variable intake ranges from 40 to 240; the maximum static pressure in the intake (40 psia) occurs at the lowest Mach number. The maximum static temperature in the intake (at Mach 26) is 2600°K. The area ratio (Fig. 3) varies from 0.11 to 0.041. At these hypersonic speeds, the position of the shock interaction point C and the reflected shock impingement points D and E are seen to be insensitive to a variation in Mach number.

Figure 4 shows that, as with ideal gases,[9] ϵ is almost equal to the difference between the aerodynamic wedge angles ($\delta_2 - \delta_1$); it is always slightly lower than the difference in the aerodynamic wedge angles. This difference increases with an increase in the δ's. For every pair of unequal aerodynamic wedge deflection angles, there exists a Mach number at which ϵ is a minimum.

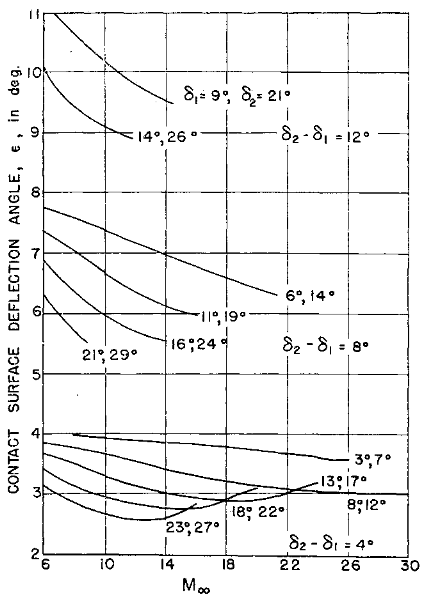

Fig. 4 Contact surface deflection angle vs M_∞ for various aerodynamic wedge deflection angles.

For angles of attack (α) up to 6°, the variation of ϵ with angle of attack α is linear and relatively insensitive to M_∞. The net flow deflection ($\epsilon - \alpha$) is always opposite to and slightly less than α, so that the pair of wedges act as a flow turning mechanism to return the flow to the axial direction as it enters the combustion chamber. It is seen that ($\epsilon - \alpha$) decreases with an increase in M_∞.

For a δ of 10°, the static temperatures at the exit of the intake are high enough to lead to spontaneous ignition of most fuels.

For hypersonic air intakes, two important parameters are the efficiency, as expressed by the nondimensional entropy increase ($\Delta S/R$), and the capability, as expressed by the kinetic energy coefficient.[6] Figure 5 shows the nondimensional entropy increase as varying from 0.5 to 4.2. This is in substantial agreement with the corresponding values given by ideal gas calculations.[6] The intake becomes less efficient but increases in capability with an increase in the angle of attack. In general, this type of intake is found to be relatively insensitive to variations in Mach number and angle of attack.

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